PROBABILITY AND STATISTICS

Probability, r.v's
r. processes
Veeraraján
McGraw Hills.

(i) Basics.

(ii) Probability

(iii) Random variable / Expectation.

(iv) Distribution → Discrete
→ Continues.

(v) Mathematical

STATISTICS

→ collection of data.
→ Analysis of data
→ Interpretation of data.

Definition: According to prof. R.A Fisher, Statistics is defined as a collection of data, analysis of data and interpretation of data.

TYPES OF DATA

(i) Grouped and Ungrouped.

(ii) closed and open data.

GROUPED DATA

If data is in the form of class intervals and frequency then the data is known as grouped data, or distributing the frequencies to their corresponding class intervals, then the data is known as Frequency distribution.

UNGROUPED DATA : If the data contains only observations, without any class intervals, then the data is known as ungrouped data or Raw Data.

CLOSED DATA : If the class intervals are in a continuous form without any discontinuity, then the data is known as closed data otherwise open data.

MEAN (AVERAGE)

$$\boxed{\bar{X}_{UGD} = \frac{\sum_{i=1}^{n} x_i}{n}}$$

$$\boxed{\bar{X}_{GD} = \frac{\sum_{i=1}^{n} f_i x_i}{N}}$$

n :→ no of observations.

x : midpoint, $\frac{UL+LL}{2}$

N : sum of frequencies.

f : frequencies.

MEDIAN

- If n is odd, the middle observation itself is the median.
- If n is even, average between the middle observations provided
  i) Data is rearranged either in increasing or decreasing order.
  ii) No: of observations above the middle is equal to the number of observations below.

$$\boxed{M_d = l + \frac{\left(\frac{N}{2} - m\right) \times c}{f}}$$

l :- Lower limit for the ideal class

f :- frequency for the ideal class.

: Cumulative frequency for the ideal class.

Q) Find the median for the following frequency data.

C.I	FREQUENCY	CUMULATIVE FREQUENCY
0-5	3	3
5-10	7	10
10-15	11	21 → Idealclass.
15-20	8	29
20-25	2	31
		N = 31

$$\frac{N}{2} = \frac{31}{2} = 15.5$$

$$M_d = 10 + \left(\frac{15.5 - 10}{11}\right)5$$

$$= 10 + \frac{5.5 \times 5}{11} = 12.5$$

Note: If the first class itself is ideal, the cumulative frequency and frequency are ideal $(m = f) \Rightarrow M_d = l$

MODE

The most frequently repeated observation is known as Mode.

1, 2, 3, 4, 5, 2, 3, 11, 14, 2, 3, 21, 2, 16, 21, 3, 19

$M_o = 2, 3 \longrightarrow$ Bimodal.

For grouped data

$$\boxed{M_o = 3\,Md - 2\,\text{Mean}}$$

$$M_o = l + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)c \qquad \Delta_1 = f - f_{-1} \qquad \Delta_2 = f - f_{+1}$$

Q Find the mode for the following frequency distribution.

C.I	Frequency
0-10	11
10-20	14
20-30	17
30-40	8
40-50	5
50-60	3

Ideal class.

Frequency 17 can be treated as Ideal class [Highest frequency]

$\Delta_1 = 17-14 = 3$

$\Delta_2 = 17-8 = 9$

$$M_o = 20 + \left(\frac{3}{12}\right) \times 10 = \frac{45}{2} = 22.5$$

* If the maximum frequencies are repeated ~~itself,~~ first, last and in between, select 'in between' as the ideal class.
* If the maximum frequencies are repeated in between, select randomly (bimodal).
* If all the frequencies are equal, mode is undefined $\left[\frac{0}{0}\text{ form}\right]$
* If the maximum frequencies are repeated first and last select randomly (bimodal).

MEASURES OF CENTRAL TENDENCIES

Among the 3 measures, mean, mode and median,
Mean is the best measure.

MEASURES OF DISPERSION

→ Range → Standard Deviation. (SD)

→ Mean Deviation. → Coefficient of Variation. (C.V)

→ Quartile Deviation (Q.D)

Measures of dispersion helps us to identify the deviation within the data.

RANGE : [Max — Min.

Greatest value — Least value]

STANDARD DEVIATION

$$\sqrt{\text{Variance}} = (S.D)$$

$$\text{Variance} = (S.D)^2 = \sigma_x^2$$

$$\boxed{\sigma_x^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n} = \frac{1}{n}\sum_{i=1}^{n}x_i^2 - (\bar{x})^2}$$

Variance is the Sum of the Squares of deviation from mean.
The differences or deviations within the data, is known as Variance.

Note:

i Lesser Variance is more consistent or more uniform.

ii Variance will never be negative.

iii Variance of constant is 0.

iv Sum of the differences from the mean is always Zero.

$$\boxed{\left[\sum_{i=1}^{n}(x_i-\bar{x})\right] = 0}$$

* If the variances are equal for the different groups, greater mean is more consistent.

* Sum of the squares of the deviation from the mean should be minimum.

For grouped data

$$\boxed{\sigma_x^2 = \frac{1}{N}\sum f_i x_i^2 - \left(\frac{1}{N}\sum f_i x_i\right)^2}$$

$$\boxed{\sigma_x^2 = \frac{1}{N}\sum f_i (x_i-\bar{x})^2}$$

GROUPED DATA VARIANCE

RELATION B/W QD/MD/SD

$$\boxed{6\,QD = 5\,MD = 4\,SD}$$

$$QD = \frac{2}{3}\sigma \quad , \quad MD = \frac{4}{5}\sigma$$

COEFFICIENT OF VARIATION (C.V)

$$\boxed{C.V = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100}$$

$$\boxed{C.V = \frac{\sigma}{\bar{x}} \times 100}$$

Lesser σ implies lesser C.V $\Rightarrow$ Data is more consistent or uniform.

For identifying the consistency within the data, which can be measurable — by standard deviation ...

Q. Find mean and variance for the first n natural numbers.

$$\bar{X} = \frac{[1+2+3+\dots+n]}{n}$$

$$= \frac{\cancel{n}(n+1)}{2\cancel{n}}$$

$$\bar{X} = \frac{n+1}{2}$$

$$\sigma_x^2 = \frac{1}{n}\sum x_i^2 - (\bar{x})^2$$

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 = \frac{1}{n}\left[1^2+2^2+3^2+\dots+n^2\right]$$

$$= \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$\sigma_x^2 = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right] = \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right] = \frac{n+1}{2}\times\frac{n-1}{6} = \frac{n^2-1}{12}$$

$$\boxed{\bar{X} = \frac{n+1}{2}}$$

Mean of n natural numbers

$$\boxed{\sigma_x^2 = \frac{n^2-1}{12}}$$

Variance of n natural numbers.

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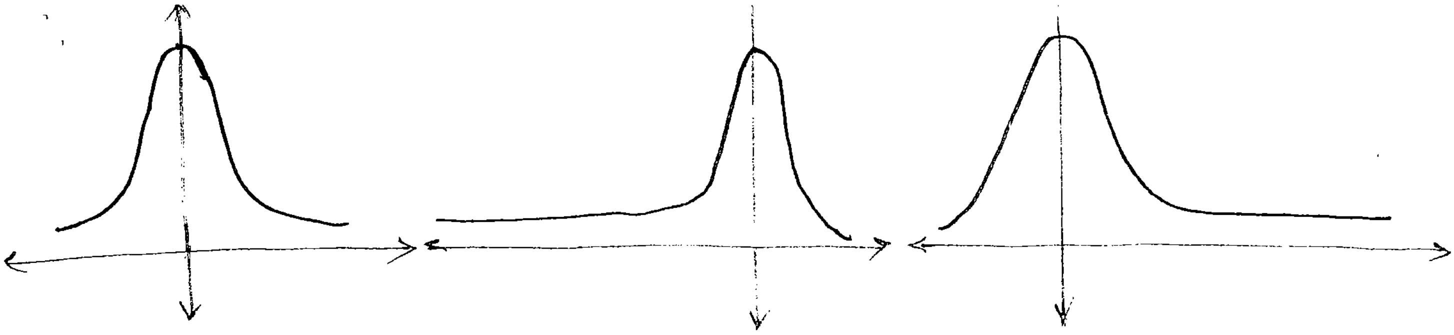
n statistics, geometrical representations or graph representations purely helps us to determine the behaviour of grouped data.)

SKEWNESS : opposite of symmetry.

"Lack of symmetry".



Symmetry. Negatively skewed. Positively skewed.

→ To check the skewness of a distribution.

→ **PEARSON'S COEFFICIENT OF SKEWNESS**

$$\boxed{S_{KP} = \frac{\text{Mean} - \text{Mode}}{\sigma}}$$

$$\boxed{S_{KP} = \frac{3(\text{Mean} - \text{Median})}{\sigma}}$$

→ practical limit of S_{KP}

$$\boxed{-3 \leq S_{KP} \leq 3}$$

→ For Symmetry $S_{KP} = 0$

Symmetry condition	Negative skewness	positive skewness.
Mode = Median = Mean	Mode > Median > Mean	Mode < Median < Mean.

PROBABILITY

RANDOM EXPERIMENT: Unpredictable outcomes of an experiment is known as a Radom experiment.

eg: Tossing a unbiased coin.

Rolling a Die.

Drawing a card from the pack of 52.

AMPLE SPACE (S): The collection of all possible outcomes of an experiment is known as a Sample space. It is dended by S.

EVENT: The outcomes of an experiment is known as a event. Mathamatically, event is the subset of Sample space.

ition of OBABILITY: The probability of an event is defined as the ratio b/w the favourable cases to the event and the number of outcomes of an experiment. (The outcomes are mutually exclusive, exhaustive events)

$$\therefore \boxed{P(E) = \frac{m}{n}} \quad \text{where } m \leqslant n$$

AXIOMATIC APROACH / PROBABILITY FUNCTION

le 1 $P(S) = 1$

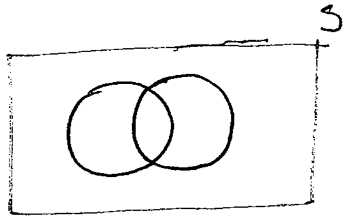
le 2 $0 < P(E) \leqslant 1$

$P(E) = 0 \;\therefore\; P(\phi) = 0 \longrightarrow$ Impossible Event

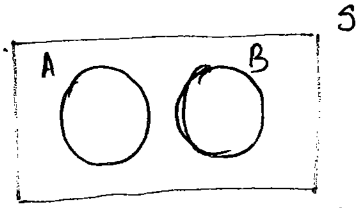
$P(E) = 1 \;\therefore\;$ — [illegible]

Rule 3 : $\boxed{P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i)}$

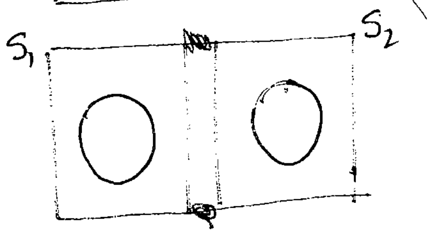
where E_i's are disjoint/Mutually Exclusive.



Dependent.



Mutually Exclusive Events



Independent

Note: Occurance of an event doesnot depends ~~on 1~~ upon the occurance of the events in the same sample space, then such events are called Mutually Exclusive event.

→ Let A and B are mutually exclusive events.

$A \cap B = \phi$ & $P(A \cap B) = 0$

→ Occurance of an event doesnot depends ~~on~~ upon the occurance of same event in a different sample space, then those events are called Independent events.

→ Mutually Exclusive events never be independent, Independent events never be ~~but~~ equal to Mutually Exclusive.

RESULTS

1. Compliment theorem.

$$P(A^c) = 1 - P(A)$$

$$P(A) = 1 - P(A^c)$$

2. Addition Theorem.

If A, & B are two events (If nothing specified, take it as ~~ind~~ dependent)

~~P(A∩~~

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

→ If A and B are mutually exclusive events

$$P(A \cup B) = P(A) + P(B)$$

$$P(A + B) = P(A) + P(B)$$

Here it doesn't mean '+' = '∪'

~~In unio~~ If '+' Sign is given it is indirectly shown or it is sure that A & B are mutually exclusive. only mutually exclusive events can be added.

$$P(A+B+C) = P(A) + P(B) + P(C)$$

3. Multiplication Theorem for Dependent Events.

If A & B are two events,

$$P(A \cap B) = P(A)\ P(B/A)$$ → Here A must be happened already.

conditional probability

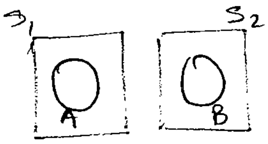
$$= P(B)\ P(A/B)$$ → Here B must be happend already.

if A, B and C are three events.

$$\boxed{P(A\cap B\cap C) = \underbrace{P(A)\,P(B/A)}_{P(A\cap B)}\;P(C/A\cap B)}$$

3. Multiplication Theorem for independent events.

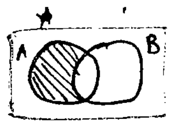
$$P(A\cap B) = P(A)\cdot P(B)$$



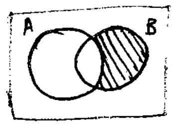
$$P(A\cap B\cap C) = P(A)\,P(B)\,P(C)$$

4. If A & B are 2 events.

$$\boxed{P(A\cap B^c) = P(A) - P(A\cap B)}$$ only A

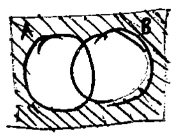


$$\boxed{P(A^c\cap B) = P(B) - P(A\cap B)}$$ only B.

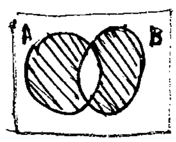


Neither A nor B

$$\boxed{P(A^c\cap B^c) = P(\overline{A\cup B}) = 1 - P(A\cup B)}$$ not in A B



$$\boxed{P(A\,\Delta\,B) = P(A\cap B^c) + P(A^c\cap B)}$$ only once



5. $$P(A^c/B) = \frac{P(A^c\cap B)}{P(B)}$$

$$= \frac{P(B) - P(A\cap B)}{P(B)} = 1 - \frac{P(A\cap B)}{P(B)} = 1 - P(A/B)$$

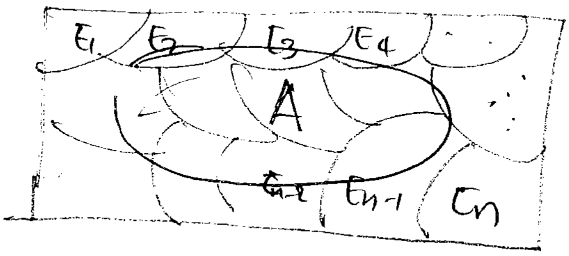
$$P\left(A/B^c\right) = \frac{P(A\cap B^c)}{P(B^c)} = \frac{P(A) - P(A\cap B)}{1 - P(B)} \quad (\because P(B) \neq 1)$$

$$P\left(A^c/B^c\right) = \frac{P(A^c\cap B^c)}{P(B^c)} = \frac{1 - P(A\cup B)}{1 - P(B)} \quad (\because P(B) \neq 1)$$

Note: If A and B are independent events, the probability of $P(A\cap B^c)$, $P(A^c\cap B)$ and $P(A^c\cap B^c)$ are also independent.

6. <u>BAYE'S THEOREM</u>

If $E_1 E_2, E_3, \ldots E_n$ are the mutually exclusive events $P(E_i) \neq 0)$ such that A is an arbitrary events which is a subset of "$\bigcup_{i=1}^{n} E_i$", then P(A) is



$$P(A) = P(E_1\cap A) + P(E_2\cap A) + \cdots + P(E_n\cap A)$$

$$P(A) = P(E_1)\,P\left(A/E_1\right) + P(E_2)\,P\left(A/E_2\right) + P(E_3)\,P\left(A/E_3\right) + \cdots + P(E_n)\,P\left(A/E_n\right)$$

$$\boxed{P(A) = \sum_{i=1}^{n} P(E_i)\,P\left(A/E_i\right)}$$

Total probability of unknown event.

Part (ii) of Bayes's theorem : Reverse probability.

$$\boxed{P\left(E_i/A\right) = \frac{P(E_i \cap A)}{P(A)}}$$

$$\boxed{P\left(E_i/A\right) = \frac{P(E_i)\, P\left(A/E_i\right)}{\sum_{i=1}^{n} P(E_i)\, P\left(A/E_i\right)}}$$

Indirectly A is known.

steps in Bayes's theorem.

S1 Identify the known events in the data (Mutually exclusive).

S2 Select the unknown event (It is a part of known events).

S3 Write the probability of unknown in terms of known.

S4 Find the total probability of unknown events.

S5 Compute Reverse probability for known events.

'Atleast'	Minimum	$\geq$
'Atmost'	Maximum	$\leq$
'And'	Product	$\cap$
'OR'	Sum	$\cup$

Application of addition theory

Cases with terminologies

→ 'either - or'

→ 'atleast once'

→ 'OR'

Application of Multiplication theory

Cases with terminologies

→ 'Simontaneously'

→ 'one after other'

→ 'as well as'

→ 'Successively'

→ 'One by one'.

→ 'alternatively'.

→ 'and'.

52 CARD CASE

Total 52 cards

13 Hearts + 13 Diamond + 13 club ~~spade~~ + 13 spade.

Each 13 contains 1, 2, 3, 4, ... 10, J, Q, K
(11) (12) (13)

ie, 10 number cards + 3 Face cards

Total ~~of~~ no: of face cards = $4 \times 3 = 12$

Here 1 sample space =

Q. 3 coins are tossed at a time. Find the probability of yielding atmost one head for

$$S = \begin{vmatrix} HHH \\ HHT \\ HTH \\ HTT \\ THH \\ THT \\ TTH \\ TTT \end{vmatrix}$$

$\cancel{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}$

$$P(X \leq 1) = P(X < 1) + P(X = 1)$$

$$= \frac{1}{8} + \frac{3}{8}$$

$$= \frac{4}{8} = \underline{\underline{\frac{1}{2}}}$$

Q. Above data same, Find the probability that atleast one tail.

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - \frac{1}{8}$$

$$= \underline{\underline{\frac{7}{8}}}$$

Q. Find the probability that atleast one head and one tails

P(atleast 1 head $\boxed{\text{and}}$ atleast 1 tail min).

→ whenever "and" is used as the conjugation b/w any number of events, the all the events must occurs simantaneously in all ~~cases~~ Favourable cases.

$$P = \frac{6}{8} = \underline{\underline{\frac{3}{4}}}$$

Q, Same data. Find the probability that atleast one head and atmost one ~~tail~~ tail.

HHH, HHT, HTH, THH $\qquad P = 4/8 = \underline{1/2}$

Q, A player tosses 4 coins Find the probability that atleast 2 heads and atleast 2 tails.

$$\frac{4P_4}{2!\,2!} = \qquad \frac{\frac{4!}{0!}}{4} \qquad \frac{4\times3\times2\times1}{4} = 6$$

HHTT
HTTH
TTHH
THHT
THTH
HTHT

$$\boxed{= 6 = {}^4C_2}$$

repeated

favourable.

$$\boxed{{}^nC_r = \frac{n!}{(n-r)!\,r!}}$$

H	No of cases	T
$^4C_0$ =	1	= $^4C_4$
$^4C_1$ =	4	= $^4C_3$
$^4C_2$ =	6	= $^4C_2$
$^4C_3$ =	4	= $^4C_1$
$^4C_4$ =	1	= $^4C_0$
	12	

favourable case.

~~Atm~~ Atmost 2 head, Atmost 2 tails = same = $\frac{6}{16}$

Q, A coin is ~~repea~~ tossed 6 times. Find the probability that the number of heads are more than the number of tails.

$$2^6 = 64$$

$$\underbrace{{}^6C_0 \quad {}^6C_1 \quad {}^6C_2 \quad {}^6C_3 \quad {}^6C_4 \quad {}^6C_5 \quad {}^6C_6}$$

Favourable Cases of Head.

$$P = \frac{{}^6C_6 + {}^6C_5 + {}^6C_4}{64}$$

$$= \frac{1 + \frac{6!}{1 \times 5!} + \frac{6!}{2!\,4!}}{64}$$

$$= \frac{1 + 6 + \frac{\cancel{6}^{3} \times 5}{\cancel{2}}}{64}$$

$$= \frac{1+6+15}{64}$$

$$= \frac{22}{64}$$

Q, A coin is repeated n times. Find the probability that the head appears in the odd terms.

$${}^nC_1 + {}^nC_3 + {}^nC_5 + \dots + {}^nC_{n-1} = 2^{n-1}$$

$${}^nC_0 + {}^nC_2 + {}^nC_4 + \dots + {}^nC_n = 2^{n-1}$$

$$\text{Req prob} = \frac{2^{n-1}}{2^n} = \underline{\tfrac{1}{2}}$$

two times.

1. Two dies are rolled, Find the probability that for getting a sum 7

(i) atleast once

(ii) only once

(iii) twice.

(i) $\underbrace{1+6,\ 5+2,\ 4+3}_{3 \times 2 = 6}$

~~$6 \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{6}$~~

sum 7 in first : $P(A) = 6/36 = 1/6$ $\quad P(A^c) = 5/6$.

sum 7 in second : $P(B) = 6/36 = 1/6$ $\quad P(B^c) = 5/6$.

$$P(\text{atleast once}) = P(A \cup B) = 1 - P(A^c \cap B^c) = 1 - P(A^c)P(B^c)$$

$$= 1 - \frac{5}{6} \times \frac{5}{6}$$

$$= 1 - \frac{25}{36}$$

$P(\text{only once})$

$$= P(A \cap B^c) + P(B \cap A^c)$$

$$= P(A)\,P(B^c) + P(B)\,P(A^c)$$

$$= \tfrac{1}{6} \times \tfrac{5}{6} + \tfrac{5}{6} \times \tfrac{1}{6}$$

$$= \frac{10}{36}$$

$$P(\text{twice}) = P(A \cap B) = P(A)\,P(B)$$

$$= \tfrac{1}{6} \times \tfrac{1}{6} = \tfrac{1}{36}$$

Q, Two dices are rolled. Find the probability that the first die should contain a prime number or a total of eight.

~~(6,2)(2,6),(5,3)(3,5),(4,4)~~

prime 2, 3, 5,

$2 \leftarrow (2,1)\ (2,2)\ \cdots\ (2,6)$

$3 \leftarrow (3,1)\ (3,2)\ \cdots\ (3,6)$

$5 \leftarrow (5,1)\ \cdots\ (5,6)$

$6 \times 3 = 18 \qquad P = \frac{18}{36}$.

Total 8

(6,2)(2,6)(5,3)(3,5)(4,4)

$P = \frac{5}{36}$

But A & B are dependent.

Hence to find $P(A \cap B)$

ie, Prime in first & sum 8

$(5,3)\ (3,5),\ (2,6) \rightarrow$ 3 cases $\quad P(A \cap B) = \frac{3}{36}$.

we need to find

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$$

$$= \frac{20}{36}$$

Two dice are rolled. Find the probability neither sum 9 nor sum 11

$$P(\text{neither sum } 9, \text{ nor } 11)$$

$$= 1 - P(\text{sum } 9, \text{ and sum } 11)$$

sum $9 \rightarrow (5,4)\ (4,5)\ (6,3)\ (3,6) \quad \rightarrow \frac{4}{36}$

sum $11 \rightarrow (5,6)\ (6,5) \quad \rightarrow \frac{2}{36}$

$$P(9^c \cap 11^c) = 1 - P(9 \cup 11)$$

$$= 1 - \left(\frac{4}{36} + \frac{2}{36}\right) = \cancel{\frac{6}{36}} \quad \frac{30}{36}$$

Q. A determinant of order 2. with the elements 0, (and) or 1. Find the probability that the choosen det is non zero

$$\begin{bmatrix} \underline{a} & \underline{b} \\ \underline{c} & \underline{d} \end{bmatrix} \quad S = 2^4 = 16$$

$\Delta \neq ad - bc$

$\therefore$ Case $\Delta = 1$ $\quad$ [a = d = 1 atleast one of b & c is zero]

$$\Delta = 1 \qquad \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$= \underline{3 \text{ cases}}$

Case II $\quad \Delta = -1$

$ad = 0 \quad bc = 1$

[b = c = 1, atleast one of a & d is zero]

$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \quad \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \Rightarrow 3 \text{ cases}.$$

$P(\text{non zero } \Delta) = 3/16 + 3/16 = \underline{6/16}$

$P(\text{non neg } \Delta) = 1 - 3/16 = \underline{13/16}$

~~$P(\text{zero } \Delta) = 1 - 6/16 = 10/16$~~

Q, 4 cards are drawn ~~from~~ at random from a ---- pack of 52 cards. Find the probability that

(i) All the 4 cards are drawn from same suit.

(ii) No two cards are drawn from the same suit.

(i) $S = {}^{52}C_4$

$$\frac{4 \times {}^{13}C_4}{{}^{52}C_4}$$ (at a time)

(ii) $$\frac{{}^{13}C_1 \, {}^{13}C_1 \, {}^{13}C_1 \, {}^{13}C_1}{{}^{52}C_4} = \frac{(13)^2}{{}^{52}C_4}$$ (one by one).

~~= 13 × 1~~

Q, A card is drawn from a pack of 52 cards. Find the probability that ~~neither a diamond nor a face car~~

(i) neither a diamond nor a face.

(ii) neither a 10 nor a king

(i) $P(D^c \cap F_c^c) = 1 - P(D \cup F)$

$P(D) = 13/52$

$P(F) = 12/52$

$$= 1 - \left[\frac{13}{52} + \frac{12}{52} - \frac{3}{52}\right] = \frac{30}{}$$ Intersection

(ii) $P(10) = \frac{4}{52}$

& $P(K) = \frac{4}{52}$

10 & King are mutually exclusive.

$$P(10^c \cap K^c) = 1 - P(10 \cup K)$$

$$= 1 - \frac{4}{52} - \frac{4}{52}$$

$$= 1 - \frac{8}{52}$$

$$= \frac{44}{52}$$

Q, A and B are the two players rolling a die on the condition that one who gets the two first wining the game. If A start the game, what are the winning chances of player A, B.

$$\frac{1}{6} + \frac{5}{6} \times \frac{1}{6} \quad \frac{5}{6} \times \frac{1}{6}.$$

$P(2) = 1/6$ $\quad P(2^c) = 5/6$

Let getting 2 is p $\quad$ not getting q.

$$P(\text{win } B) = \cancel{q+p}\ qp + q^3p + q^5p + \dots = qp + q^2\, qp +$$

$$= p\left[q + q^3 + q^5 + \dots\right] = pq\left[\cancel{q}\ 1 + q^2 + q^4 + \dots\right]$$

$$= (qp) \times \frac{1}{(1-q^2)}$$

$$= \frac{1}{6} \times \frac{5}{6} \times \frac{1}{1-\frac{25}{36}}$$

$$= \underline{\underline{5/11}}$$

$$P(\text{win } A) = p + q^2 p + q^4 p + \dots$$

$$= p\left[1 + q^2 + q^4 + \dots\right]$$

$$= p \times \frac{1}{1-q^2} = \frac{1}{6} \times \frac{1}{1-\frac{25}{36}}$$

$$= \frac{1}{\cancel{6}} \times \frac{\cancel{36}\,6}{11}$$

$$= \underline{\underline{\frac{6}{11}}}$$

A, B, C are the 3 players in the order. Tossing the same coin on the condition that one who gets the head first winning game. If A starts the game, what are the winning chances of player C in 3rd trial.

$$P(\text{win } C \text{ in } 3^{rd} \text{ trial}) = q^3 q^3 q^2 p$$

$P(H) = 1/2$ 'p' $P(T) = 1/2$ (q)

$$= \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 \times \frac{1}{2}$$

$$= \frac{1}{8} \times \frac{1}{8} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{512}$$

Q A number is choosen at random from 100 numbers,

$\{ (00)\ 01, 02, 03, \ldots 99 \}$

Let x denote the sum of the digits on the number, and y denotes product of the digits on the number. Find the probability that

$$P\left(X=9 / Y=0\right) = \frac{P(X=9 \cap Y=0)}{P(Y=0)}$$

$\underline{0}\ \underline{0} - 9$

$$P(Y=0) = \frac{2/100}{19/100} = \underline{\underline{2/19}}$$

9 ~~10~~ + 10 $\underline{\ }\ \underline{0}$.

Q, 60% of the employees of the company are college graduates. of these, 10% are in the sales department. of the employees who did not graduate from the college are 80% in the sales department. A person is selected at random. Find the probability that

(i) The person is in the sales department.

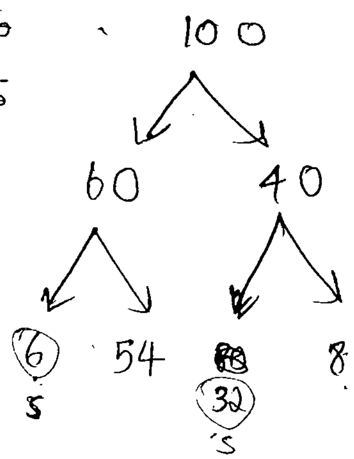
(ii) Neither in the sales department nor a college ~~departs~~ graduate.

$60 \times \frac{10}{100}$

$40 \times \frac{80}{100}$

(i) $\frac{38}{100}$ ~~P(CG ∩ SD)~~

(ii) $\frac{8}{100}$



(i) $P(SD) = P(CG \cap SD) + P(CG^c \cap SD)$

$$= P(CG)\, P(SD/CG) + P(CG^c)\cdot P(SD/CG^c)$$

$$= 0.6 \times 0.1 + 0.4 \times 0.8$$

$$P(SD) = \underline{\underline{0.38}}.$$

(ii) $P(CG^c \cap SD^c) = 1 - P(CG \cup SD)$

$$= 1 - [P(CG) + P(SD) - P(CG \cap SD)]$$

$$= 1 - [P(CG) + P(SD) - P(CG)\, P(SD/CG)]$$

$$= 1 - [0.6 + 0.38 - 0.6 \times 0.1]$$

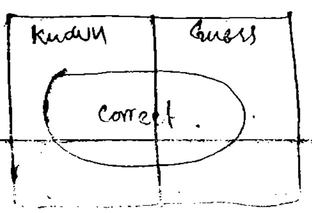
$$= 1 - 0.92$$

$$= \underline{\underline{0.08}}$$

Q. In answering a qn on multiple choise qn, a student either knows the answer or guess the answer. Let P be the probability that student knowing the answer to the qn and (1-P) be the probability that guessing the ans to ~~a~~ qn. Assume that if the student guess the answer to ~~the~~ a question will be correct, with probability 1/5. what is the conditional probability that if the student knew the ans to a qn, given that, he answered correctly?

$P(K) = P$

$P(G) = 1-P$



E: answering correctly.

$P(K) = P$

$P(G) = 1-P$

$$P(E) = P(E\cap K) + P(E\cap G)$$

$$P(E) = P(E\cap K) + \cancel{P(G)\,P(E/G)}\; P(E\cap G)$$

$$P(E) = P(K)\,P(E/K) + P(G)\,P(E/G)$$

$$= P\,P(E/K) + (1-P)\,1/5$$

$P(E/K) = 1$

if Knowing correct then he will correct it for sure.

$$= P\times 1 + (1-P)\,1/5$$

$$= P + 1/5 - P/5$$

$$P(E) = \frac{4P+1}{5}$$

Inverse probability

$$P(K/E) = \frac{P(K\cap E)}{P(E)}$$

$$= \frac{P\times 1}{\frac{4P+1}{5}} = \frac{5P}{4P+1}$$

In qn. final known event → correct ans.
unknown event → Knowing ans.
Hence go for Reverse probability.

Q. There are 3 coins. Of these two are unbiased. One is a biased coin with 2 heads. A coin is drawn at random and tossed two times. It appears head on both the ~~side~~ times, Find the probability that it is from the biased coin.

UB   B

2   1   = 3

$P(UB) = 2/3 \qquad P(B) = 1/3$.

E: Getting a head two times. is an unknown event.

$$P(E) = P(E \cap UB) + P(E \cap B)$$

$$\cancel{= P(UE)\ P(UB/E) + P(}$$

$$= P(UB)\ P(E/UB) + P(B)\ P(E/B)$$

$P(E/B) = 1$

$$= 2/3 \times \frac{1}{4} + 1/3 \times 1$$

$P(E/UB) = \frac{1}{2} \times \frac{1}{2} = 1/4$

$$= \frac{1}{6} + \frac{1}{3}$$

$$P(E) = \frac{3}{6}$$

an: $P(B/E) = \dfrac{P(E \cap B)}{P(E)}$

$$= \frac{P(B)\ P(E/B)}{3/6} = \frac{1/3 \times 1}{3/6} = \frac{\cancel{6}^{2}}{\cancel{9}3} = \frac{2}{3}$$

Ans = 2/3

1. player A speaking truth 4 out of 7 times, A card is drawn from the pack of 52 cards. He reports that there is a diamond what is the probability that actually there was a diamond.

$\frac{4}{7}$

T: A Player telling truth

~~$P(A) = 4/7$~~

$P(T) = (4/7)$

Lie probability

$P(L) = (3/7)$

D: Reporting a diamond.

$$P(D) = P(T \cap D) + P(L \cap D)$$

$$= P(T)\,P(D/T) + P(L)\,P(D/L)$$

$$= \frac{4}{7} \times \cancel{13}\,\frac{1}{4} + \frac{3}{7} \times \frac{3}{4}$$

$$= \frac{4+9}{28}$$

$$= \frac{13}{28}$$

$P(D/T) = 13/52$

$P(D/L) = 39/52$

~~$P(D/T) = P(1$~~

$$P(T/D) = \frac{P(T \cap D)}{P(D)}$$

$$= \frac{\cancel{\frac{4}{7} \times \frac{1}{4}}}{\frac{13}{\cancel{28}}} = \frac{4}{13}$$

Q. A letter is known to have come from either Tatanagar or CALCUTTA ~~Calcutta~~. On the envelope, the just two consecutive letters 'TA' are visible. Find the probability that the letter has come from Tatanagar.

~~2/3~~

$P(T) = 1/2 \quad : \quad P(C) = 1/2$.

E : Getting a TA consider as single letter.

$P(E/T) = 2/8$

$P(E/C) = 1/7$

$$P(E) = P(E \cap T) + P(C \cap E)$$
$$= P(T)\,P(E/T) + P(C)\,P(E/C)$$
$$= 1/2 \times 2/8 + 1/2 \times 1/7$$
$$= 11/56$$

Qn $P(T/E) = \dfrac{P(T \cap E)}{P(E)} = \dfrac{1/2 \times 2/8}{11/56} = \dfrac{7}{11}$.

Given only ~~@~~ 1 letter is visible (which is a combination of 2 consecutive) Not more than 1 is visible.

1 2 3 4 5 6 7 8
TATANAGAR

CALCUTTA
1 2 3 4 5 6 7

Hence can take either one of 'TA' in TATANAGAR. No need of confusion.

There are 3 bags A, B, C, with Balls Blue, Red, and Green in the form ~~of 1, 2, 3~~

~~B R G~~

		colour	
	B	R	G
A	1	2	3
Bags. B	2	3	1
C	3	1	2

A bag is drawn at random, and two balls are taken from it. They are found to be one blue and one red. Find the probability that the selected balls are from bag C.

$$P(A) = P(B) = P(C) = 1/3 .$$

E : Getting one blue and one red.

$$P(E) = P(E\cap A) + P(E\cap B) + P(E\cap C)$$

$$= P(A)\,P(E/A) + P(B)\,P(E/B) + P(C)\,P(E/C)$$

$$= \frac{1}{3}\left[\frac{2}{15} + \frac{6}{15} + \frac{3}{15}\right]$$

$$= \frac{1}{3}\left[\frac{11}{15}\right]$$

$$= \frac{11}{45}$$

$$P(E/A) = \frac{{}^1C_1 \times {}^2C_1}{{}^6C_2} = \frac{2}{15}$$

$$P(E/B) = \frac{{}^2C_1 \times {}^3C_1}{{}^6C_2} = 6/15$$

$$P(E/C) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{3}{15}$$

Qn: $P(C/E) = \frac{P(C\cap E)}{P(E)}$

$= \frac{1}{3} \times \frac{3}{15}$ $\quad \frac{3}{11}$

Also others can be found out.

$$P(B/E) = \frac{\frac{1}{3} \times \frac{6}{15}}{\frac{11}{45}} = \frac{6}{11}$$

$$P(A/E) = \frac{\frac{1}{3} \times \frac{2}{15}}{\frac{11}{45}} = \frac{2}{11}$$

LEAP YEAR CONCEPT

LEAP YEAR

366 Days

52 weeks + 2 days

S-M
M-T
T-W
W-T
T-F
F-S
S-S

$P(53 \text{ sunday}) = \frac{2}{7}$

NON LEAP YEAR.

365 Days.

52 weeks + 1 day

$P(53 \text{ sundays}) = \frac{1}{7}$

Two dice.

$$P(\text{diff zero}) = P(\text{Doublets}) = \frac{6}{36} = \frac{1}{6}$$

$P(1,1)$

$P(2,2)$

$\vdots$

$P(6,6)$

Three dice

$$P(\text{Triplet}) = \frac{6}{6^3} = \frac{1}{36}$$

$(1,1,1)$

$(2,2,2)$

$\vdots$

$(6,6,6)$

Consider now

$\{1, 2, 3, \ldots, 200\}$

$P(\text{div } 6 \text{ OR div by } 8)$

$$P(\text{div } 6) = \frac{33}{200}$$

$$P(\text{div } 8) = \frac{25}{200}$$

$$P(\text{div } 6 \text{ AND div } 8) = \text{LCM}(6,8)$$

$$= P(\cancel{24} \text{ div by } 24) = \frac{8}{200}$$

Hence $P(\text{div by } 6 \text{ OR div by } 8) = P(6) + P(8) - P(24)$

$$= \cancel{25}\ \frac{33}{200} + \frac{25}{200} - \frac{8}{200}$$

$$= \frac{50}{200} = \frac{1}{4}$$

RANDOM VARIABLE AND EXPECTATION

(r.v)

RANDOM VARIABLE : Connecting the outcomes of an experiment with real values is known as Random Variables.

(It is a rule to assign Real number to the outcome is known as 1D Random Variable)

The corresponding data is known as univariate data.

2-D RANDOM VARIABLE : Connecting 2 outcomes at a time to the one real value provided those two outcomes are drawn from same sample space.

The corresponding data is known as Bivariate data.

⇒ Similarly the concept of n-D Random variable which corresponds to an n-tuple.

TYPES OF RANDOM VARIABLE

Random Variable

- Discrete Random Variable
  - Probability MASS FUNCTION — $P(x)$
    - DISCRETE PROBABILITY DISTRIBUTION
      - Binomial
      - Poisson
- Continous Random Variable
  - Probability DENSITY FUNCTION — $f(x)$
    - CONTINUOUS PROBABILITY DISTRIBUTION
      - Normal
      - Uniform.

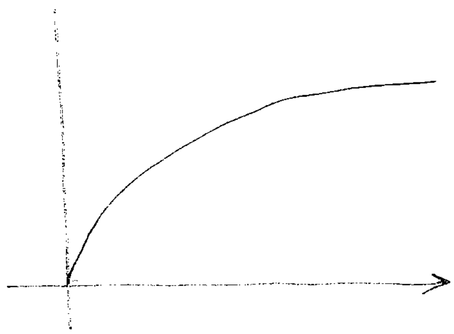
Probability Mass Function $\longrightarrow$ $P(x)$

Probability Density Function $\longrightarrow$ $f(x)$

Distributive Function/Cumulative Function $\longrightarrow$ $F(x)$

$$\frac{dF(x)}{dx} = f(x), \qquad F(x) = \int^{x} f(x)\, dx.$$

Distribution function graph will always be a non decreasing function.



RANDOM PROCESS : Random variable along with time domain.

EXPECTATION

It is actually the mean in the probability ~~function~~ distribution.

$$E(X) = \sum_{x=0}^{n} x \cdot P(x) \qquad \text{where, } x \text{ is Discrete r.v}$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx \qquad \text{where } x \text{ is continuous r.v}$$

The above relations are derived from Frequency distribution where freq is replaced by probability

$$\bar{X} = \frac{\sum f \cdot x}{\sum f}$$

$$\bar{X} = \frac{\sum P(x) \cdot x}{\sum P(x)}$$

But $\sum P(x) = 1$

$$\therefore \bar{X} = \sum x\, P(x) = E(x)$$

(ii) VARIANCE

From frequency distribution, the variance is given by .

$$\sigma^2 = \frac{1}{N}\sum f_i x_i^2 - \left(\frac{1}{N}\sum f_i x_i\right)^2$$

$$\frac{1}{N}\sum f_i x_i = E(x)$$

$$\frac{1}{N}\sum f_i x_i^2 = E(x^2)$$

In general $\boxed{\frac{1}{N}\sum f_i x^r = E(x^r)}$

In probability distribution,

$$\boxed{V(x) = E(x^2) - (E(x))^2}$$

$$\boxed{V(x) = E\left(x - (E(x))^2\right)}$$

$$\boxed{V(x) = \sum x^2 p(x) - \left(\sum x \cdot p(x)\right)^2}$$ where x is a discrete r.V

$$\boxed{V(x) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left(\int x f(x)\,dx\right)^2}$$ where x is a continuous r.V .

PROPERTIES OF EXPECTATION

(i) If X is a r.v and 'a' is a constant,

then $E(aX) = aE(X)$

If X and Y are r.v's.

(ii) Then $E(X+Y) = E(X) + E(Y)$

$E(X-Y) = E(X) - E(Y)$

(iii) If X & Y are r.v's

$E(X.Y) = E(X) . E(Y/X)$

$= E(Y) . E(X/Y)$

(iv) If X & Y are Independent random variables,

$E(X \cdot Y) = E(X) . E(Y)$

(v) If $Y = aX + b$, where a & b are constants,

Then $E(Y) = aE(X) + b$

(vi) ie, $\boxed{E[\text{constant}] = \text{constant}}$ ie, Mean of constant = That constant itself.

(vii) $\boxed{E\left[E\left[E(X)\right]\right] = \text{constant} = E(X)}$

PROPERTIES OF VARIANCE

i) If X is a r.v and 'a' is a constant,

$$V(aX) = a^2 V(x)$$

$$V(-Y) = (-1)^2 V(Y) = V(Y)$$

If X and Y are independent r.V's.

$$V(X+Y) = V(X) + V(Y)$$

$$V(X-Y) = V(X) + V(-Y)$$

$$\Rightarrow V(X-Y) = V(X) + V(Y)$$

$$\Rightarrow \boxed{V(X \pm Y) = V(X) + V(Y)}$$

If a & b are constants, X & Y are independent r.v's,

$$V(aX - bY) = a^2 V(X) + b^2 V(Y)$$

$$V\left(X/a - Y/b\right) = 1/a^2\, V(X) + 1/b^2\, V(Y)$$

f $Y = aX + b$, where a & b are constants,

$$V(Y) = V(aX + b)$$

$$= V(aX) + V(b)$$

$$V(Y) = a^2 V(X) + 0$$

i.e. $V[\text{Constant}] = 0$

If X and Y are two random variables (Dependent r.v's).

$$\boxed{V(X+Y) = V(X) + V(Y) + 2\,CoV(X,Y)}$$

where $CoV(X,Y) \rightarrow$ Covariance of X, Y

where

$$\boxed{CoV(X,Y) = E(X\cdot Y) - E(X)\cdot E(Y)}$$

$\boxed{V(X,Y) \text{ is meaningless}}$

$\rightarrow \boxed{CoV(X,X) = V(X)}$

$$\rightarrow CoV(a,b) = E(a\cdot b) - E(a)E(b)$$
$$= ab - a\cdot b$$
$$= 0$$

$\boxed{CoV[a,b] = 0}$ where a and b are constants.

1 If x and y are independent r.v, then covariance of X, Y = 0

$CoV(X,Y) = 0$ X, Y are independent

But Converse of the statement is not true.

2. Variance and covariance are independent of change of ~~origin, dependent of change of scale.~~

$\therefore V[aX+b] = a^2 V[x]$

: Mean [Expectation] dependent of origin as well as Dependent of change of scale.

$$E[aX+b] = a^2 V[x] + b$$

SKEWNESS

$$\boxed{\beta_1 = \frac{\mu_3^2}{\mu_2^3}}$$

$\mu_3 \rightarrow 3^{rd}$ central moment.

$\mu_2 \rightarrow$ Variance.

Skewness is defined in terms of 'μ_3'

$$\boxed{\gamma = \sqrt{\beta_1}}$$

γ is also a measure of skewness.

Note :

If $\mu_3 = 0 \Rightarrow \beta_1 = 0$ Then the curve is SYMMETRY

If $\mu_3 \rightarrow -ve \Rightarrow$ Then the curve is NEGATIVELY SKEWED

If $\mu_3 \rightarrow +ve \Rightarrow$ Then the curve is POSITIVELY SKEWED

Q. Find the expectation of the number on a die when it is ~~throug~~ thrown.

$$E(X) = \sum_{0}^{n} x\, p(x)$$

$$= 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6}$$

$$= \frac{1}{6}\left[1+2+3+4+5+6\right]$$

$$= \frac{1}{\cancel{6}}\left[\frac{\cancel{6}\times 7}{2}\right]$$

$$= \cancel{2.5}\ \underline{3.5}$$

x r.v	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

Q. Find the variance for the single die

$$V(X) = \cancel{E(X)}\ E(X^2) - (E(X))^2$$

$$E(X^2) = \sum_{0}^{n} x^2\, p(x)$$

$$= \frac{1}{6}\left[1^2+2^2+3^2+4^2+5^2+6^2\right]$$

$$= \frac{\cancel{6}(7)(13)}{6\,\cancel{6}}$$

$$= \frac{91}{6}$$

$$\frac{13}{7}$$

$$\therefore V(X) = \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{91}{6} - \frac{49}{4} = \frac{182-147}{12} = \frac{35}{12}$$

Note:

The mean and variance for the sum of the numbers on the ~~dies~~ dice is

$\boxed{E(X) = \frac{7n}{2}}$ $\boxed{V(X) = \frac{35}{12}n}$ where 'n' is the number dice rolled

Q 3 unbiased dice are thrown. Find the mean ~~of~~ and variance for the sum of the numbers on them.

X : sum of number on 3 dies.

X : 2, 3, 4, . . . , 18

$$E(X) = \frac{7n}{2} = \frac{7\times 3}{2} = \frac{21}{2}$$

$$V(X) = \frac{35}{12}n = \frac{35\times 3}{12} = \frac{35}{4}$$

Two unbiased dies are rolled. Find the expectation for sum 7 on them.

X : sum for the number obtained on 2 dies.

Here X is assuming only 7.

Hence no need of additions.

$$E(\text{sum } 7) = 7 \cdot P(7)$$

$$= 7 \cdot \frac{6}{..}$$

6 cases $\begin{cases} 1, 6 \\ 6, 1 \\ 3, 4 \\ 4, 3 \end{cases}$

Q. A player tosses 3 coins. He wins 500 rupees if 3 heads occur, 300 rs if 2 Heads occurs, 100 rs if only 1 head occur. On the other hand he looses 1500 rs If 3 tails occurs Find value of the game.

X : No :of head possibility

~~$E(X) = 500 \times \frac{1}{8} +$~~

X	3	2	1	0
P(x)	1/8	3/8	3/8	1/8

$\boxed{\text{VALUE OF GAME} = \text{GAIN} - \text{LOSE} = \text{Gain} \times \text{its prob} - \text{Lose} \times \text{its prob.}}$

$$E(X) = 500 \times \frac{1}{8} + 300 \times \frac{3}{8} + 100 \times \frac{3}{8} - 1500 \times \frac{1}{8}$$

$$= \frac{1400 + 300 - 1500}{8}$$

$$= \frac{1700 - 1500}{8}$$

$$= \frac{200}{8} = 25$$

NOTE :

If Game is said to be Fair, the expected value of game is said to be 0. (no Loss and no Gain).

Q. A man has given n keys of which one fits the lock. He tries them successively without replacement to open the lock. What is the probability that the lock will be open at the r^{th} trial. Also determine mean and variance.

Note: with Replacement implies that it is Independent events.

without Replacement implies that it is Dependent Events.

prob of opening the lock in 1^{st} trial $= 1/n$

" " 2^{nd} trial $= 1/(n-1)$

" " 3^{rd} trial $= 1/(n-2)$

$\vdots$

prob of opening lock 1^{st} success in 2^{nd} trial $= \left(1 - 1/n\right)\frac{1}{n-1}$

$$= \frac{\cancel{n-1}}{n} \times \frac{1}{\cancel{n-1}} = \frac{1}{n}$$

prob of opening lock in 1^{st} success in 3^{rd} trial $= \left(1 - 1/n\right)\left(1 - 1/(n-1)\right) \times \frac{1}{n-2}$

$$= \frac{\cancel{n-1}}{n} \times \frac{\cancel{n-2}}{\cancel{n-1}} \times \frac{1}{\cancel{n-2}}$$

$$= \underline{1/n}$$

Hence

prob of opening lock 1^{st} success r^{th} trial $= 1/n$

Variance of n natural numbers $V(x) = \frac{n^2-1}{12}$

Consider a value eg:

If Keys numbered from 100 — 999

Prob (1st success in 450th trial without replacement) $= \frac{1}{900}$ (100–999 means 0→900)

Prob (1st success in 450th trial with replacement) $= \left(1 - \frac{1}{900}\right)^{449} \frac{1}{900}$

ie in without replacements, they are dependent, $\cancel{\frac{900}{}}\ \frac{\cancel{899}}{900} \times \frac{\cancel{898}}{\cancel{899}} \times \frac{\cancel{897}}{\cancel{898}} = \frac{1}{900}$

~~they cancel~~ they loss probabilities of $(r-1)$ trials cancel each other and get only $\frac{1}{900}$.

But in with replacement, each trial is independent event, Hence each ~~loss prob~~ of $(r-1)$ loss probabilities we need to multiply

$\therefore \; q^{r-1} p$

Note: The probability for the r^{th} success in the r^{th} trial with replacement is

$$\boxed{P(1^{st} \text{ success in } r^{th} \text{ trial with replacement}) = q^{r-1} p}$$

$q \rightarrow$ Failure prob

$p \rightarrow$ Success prob

Q. if x is said to be a Continuous variable and its probability function ϕ

$$f(x) = kx^2 \quad 0<x<1$$

(i) Find the value of k.

(ii) Find Mean & variance.

(i) ~~$\Sigma f()$~~ $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^1 kx^2\,dx = 1$$

$$k\left[\frac{x^3}{3}\right]_0^1 = 1$$

$$\frac{k}{3} = 1$$

$$\underline{\underline{k = 3}}$$

(ii) Mean $= E(X) = \int_{-\infty}^{\infty} x\,f(x)\,dx$

$$= 3\int_0^1 x^3\,dx$$

$$= 3\left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{3}{4}$$

(iii) Variance $V(x) = E(x^2) - (E(x))^2$

$$E(x^2) = \int_0^1 x^2 f(x)\, dx = 3\int_0^1 x^4\, dx = \underline{\underline{\frac{3}{5}}}$$

$$\text{Variance} = \frac{3}{5} - \left(\frac{3}{4}\right)^2$$

$$= \frac{3}{5} - \frac{9}{16}$$

$$= \frac{48-45}{80} = \underline{\underline{\frac{3}{80}}}$$

Q. If x is a continous r.v, and $f(x) = kx^2 e^{-x}$, ~~where~~ $0 < x < \infty$

(i) Find the value k

(ii) Mean & variance.

(i) $$\int_{-\infty}^{\infty} f(x)\, dx = 0 \qquad \int_0^{\infty} kx^2 e^{-x}\, dx = 1$$

Use Gamma Function,

$$\Gamma n = \int_0^{\infty} e^{-x} x^{n-1}\, dx \quad , n > 0$$

$$\Gamma n = (n-1)\Gamma n-1$$

$= (n-1)!$ (only when n is a integer)

$\Gamma 1 = 1$

$\Gamma 0$ = doesnot exist.

So integral becomes.

$$K\,\Gamma(3) = 1$$

$$K\,2! = 1$$

$$K = \tfrac{1}{2}$$

(ii) Mean

$$E(X) = \int_0^\infty x\, f(x)\, dx$$

$$= \frac{1}{2}\int_0^\infty x\; x^2 e^{-x}\, dx$$

$$= \frac{1}{2}\int_0^\infty x^3 e^{-x}\, dx$$

$$= \frac{1}{2}\Gamma(4)$$

$$= \frac{1}{2}\times 3! = 3$$

Variance,

$$E(X^2) = \int_0^\infty x^2 f(x)\, dx$$

$$= \frac{1}{2}\int_0^\infty x^2 \cdot x^2 e^{-x}\, dx$$

$$= \frac{1}{2}\int_0^\infty x^4 e^{-x}\, dx$$

$$= \frac{1}{2}\Gamma(5)$$

$$= \frac{1}{2}\,4! = 12$$

$$\text{Variance} = 12 - 3^2$$
$$= 12 - 9 = \underline{3}$$

Q. $f(x) = |x| \quad -1 < x < +1$

$V(x)$ Find variance.

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x) = \int_{-1}^{1} x f(x)\, dx = \int_{-1}^{1} x|x|\, dx.$$

$$= -\int_{-1}^{0} x^2\, dx + \int_{0}^{1} x^2\, dx$$

$$= \underline{\underline{0.}}$$

odd fn x even fn
= odd fn

$\int_{-a}^{a} \text{odd fn } dx = 0$

$$E(x^2) = \int_{-1}^{+1} x^2 f(x)\, dx$$

$$= \int_{-1}^{+1} x^2 |x|\, dx$$

$x^2|x| \rightarrow$ even x even
$\Downarrow$
even

$\int_{-a}^{a} \text{even fn} = 2\int_{0}^{a} \text{even fn.}$

$$= 2\times\int_{0}^{1} x^3\, dx$$

$$= 2\times\left[\frac{x^4}{4}\right]_0^1$$

$$= 2/4 = 1/2$$

$$\text{Variance} = 2/4 - 0^2 = \underline{1/2}$$

Q, If X & Y are the r.v's mean of X is 10, Variance of X = 25. Find positive values of a, b, such that Y= aX−b has Expectation is zero and Variance is 1.

$$E(Y) = E[aX-b] = 0$$

$$aE[X] - E[b] = 0$$

$$a \times 10 - b = 0$$

$$10a - b = 0$$

$$\underline{\underline{b = 10a}}$$

$$V[Y] = V[aX-b] = a^2 V[X] = 1$$

$$= a^2 \, 25 = 1$$

$$\underline{\underline{a = \frac{1}{5}}} \quad \text{Given +ve}$$

$$b = 10a$$

$$= 10 \times \frac{1}{5}$$

$$= \underline{\underline{2}}$$

$$\underline{\underline{a = 1/5,}} \quad b = 2$$

BIVARIATE DATA

Let x, y be two discrete r.v's,

Their probability together given by Joint probability Mass Function (JPMF)

Let x, y be two continous r.v's

Their probability together given by Joint Probability Density Function (JPDF)

Case (i) Continous R.V's.

→ If x and y are two continous r.v's, and its probability function is known as Joint probability density function is denoted by $f(x,y)$.

→ The marginal density functions are

$$f(x) = \int_y f(x,y)\,dy$$

$$f(y) = \int_x f(x,y)\,dx$$

→ If x and y are 2-D continous Independent r.v's ~~and its probability function is known as Joint~~ Iff,

$$\boxed{f(x,y) = f(x)\,f(y)}$$

ie,

$$\boxed{JPDF = MDf(x) \cdot MDf(y)}$$

Joint Distribution Function ~~F~~ or Cumulative Distribution Function

~~$F(x,y)$ is given by JDF~~

$$\frac{d^2}{dx\,dy} F(x,y) = f(x,y)$$

$$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{x} f(x,y)\,dx\,dy.$$

⇒ Conditional probability

$$f(x/y) = \frac{f(x,y)}{f(y)}, \quad (f(y) \neq 0)$$

$$E(x/y) = \frac{E(x,y)}{E(y)}, \quad (E(y) \neq 0)$$

Case (ii) Discrete R.V's.

If x and y are two dimensional r.v's and its probability function is known as joint probability function ~~PQ~~ is denoted by $P(x,y)$.

The marginal mass functions are

$$P(x) = \sum_{y} P(x,y)$$

$$P(y) = \sum_{x} P(x,y)$$

Q, If x and y are 2-D continous r.v's and its probability function is given by

$$f(x,y) = x \cdot y$$

$$0 < x < 1$$

$$0 < y < 1$$

(i) $E(X)$; $V(Y)$

(ii) $E(xy)$; $\mathrm{cov}(X,Y)$

(iii) $f(x/y)$; $E(X/y)$

(iv) check X & Y are independent or not

(i) $f(x) = \int_y f(x,y)\,dy = \int_0^1 x.y\,dy = x \cdot y^2/2 \Big|_0^1 = \frac{x}{2}$

$f(y) = \frac{y}{2}$

$E(x) = \int_0^1 x.f(x)\,dx = \int_0^1 x^2/2\,dx = \left[\frac{x^3}{6}\right]_0^1 = \underline{\underline{1/6}}$

$E(y) = \int_0^1 y\,f(y)\,dy = \int_0^1 \frac{y^2}{2}\,dy = \left[\frac{y^3}{6}\right]_0^1 = \underline{\underline{1/6}}$

$E(y^2) = \int_0^1 y^2 f(y)\,dy = \int_0^1 y^2 . y/2\,dy = \underline{\underline{1/8}}$

$V(Y) = \frac{1}{8} - \left(\frac{1}{6}\right)^2 = \frac{1}{8} - \frac{1}{36} = \frac{36-8}{8 \times 36} = \frac{28}{8 \times 36} = \underline{\underline{7/72}}$

$$E(x.y) = \int_{x=0}^{1}\int_{x=0}^{1} x.y\ f(x,y)\ dx\,dy.$$

$$= \int_{x=0}^{1}\int_{y=0}^{1} x.y\ (x.y)\ dx\ dy$$

$$= \int_{x=0}^{1}\int_{y=0}^{1} x^2 y^2\ dx\ dy$$

$$= \int_0^1 \frac{x^2}{3}\ dx = \frac{1}{9}$$

Covariance.

$$\text{Cov}(X,Y) = \cancel{E(X}\ E(X.Y) - E(X).E(Y)$$

$$= 1/9 - 1/6 \times 1/6 = 1/9 - 1/36 = \frac{4-1}{36} = \frac{3}{12}$$

~~(iv)~~ (iii)

$$f(x/y) = \frac{f(x,y)}{f(y)} = \frac{x.y}{y/2} = 2x$$

$$E(X/Y) = \frac{E(x.y)}{E(y)} = \frac{1/9}{1/6} = 2/3$$

$$f(x,y) = f(x)\,f(y)$$

$x.y \neq \frac{x}{2}.\ y/2$ $\therefore$ x & y are Dependent r.v's.

Q If x and y are 2-D Discrete r.v's and its joint probability mass function is

$X \backslash Y$	-1	0	$+1$
-1	$1/4$	0	0
0	0	$1/2$	0
$+1$	0	0	$1/4$

(i) Find $P\left(X+Y=2 / X-Y=0\right)$

$$P\left(X+Y=2 / X-Y=0\right) = \frac{P(X+Y=2 \cap X-Y=0)}{P(X-Y=0)}$$

$$= \frac{P(X=1, Y=1)}{P(X=-1, Y=-1) + P(X=0, Y=0) + P(X=1, Y=1)}$$

$$= \frac{1/4}{1/4 + 1/2 + 1/4}$$

$$P\left(X+Y=2 / X-Y=0\right) = \underline{\underline{1/4}}$$

BINOMIAL DISTRIBUTION

Definition : If x is said to be a binomial random variable. It allows the values from 0 to n with the parameters (n, p) and its probability mass function is

$$B(x, n, p) = p(x) = \begin{cases} {}^{n}C_{x}\, p^{x} q^{n-x}, & 0 \leq x \leq n. \quad p+q = 1, \quad q = 1-p \\ 0, & \text{otherwise} \end{cases}$$

Conditions

- Observations are independent, (n is small).
- The probability of success is constant (p is large)
- Mean is greater than the variance.

PROPERTIES

$$E(x) = \text{Mean} = np$$

$$V(x) = \mu_2 = npq$$

3rd central : $\mu = npq(q-p)$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{n^2p^2q^2(q-p)^2}{n^3p^3q^3}$$

$$\beta_1 = \frac{(1-2p)^2}{npq}$$

$$\gamma_1 = \frac{1-2p}{\sqrt{npq}}.$$

$\therefore$ In het, $p = 1/2 \longrightarrow$ symmetry.

~~PD~~ $p < 1/2 \longrightarrow$ positively skewed.

$p > 1/2 \longrightarrow$ Negatively skewed.

<u>Moment Generating Function (MGF)</u>

$$\boxed{M_x(t) = E\left[e^{tx}\right] = (q+pe^t)^n}$$

$\Rightarrow$ ~~Momentu~~ Moment Generating Function is used for the taking the sum and differente of 2 r.v's ~~tw~~ along with the Binomial Distribution.

<u>Characteristic Function.</u>

$$\boxed{\phi_x(t) = E\left[e^{itx}\right] = (q+pe^{it})^n}$$

---

characteristic ~~equation~~ is used for finding convolution and ratio of 2 r.v's with Binomial distribution.

Note:

→ $P = \frac{1}{2} \Rightarrow \mu_3 = 0 \Rightarrow \beta_1 = 0$

Then the Curve is Symmetry.

→ If $P < \frac{1}{2}$, then the curve is Positively skewed.

If $P > \frac{1}{2}$, then the curve is Negatively skewed.

The moment Generating function is used to find addition and differences b/w the r.v's with their corresponding probability function.

The characteristic function is used to finding the convolution and ratio b/w the r.v's with their probability function.

Sum of Independent Binomial r.v's is also a Binomial random variables.

Q, Find the probability of getting a 9 exactly 2 in 3 times with a pair of dice.

$n = 3$

$X = 2$

$P(\text{sum } 9) = (5,4)\,(4,5)\,(6,3)\,(3,6)$

$= \frac{4}{36} = 1/9$

$q = 8/9$

required prob $= {}^{n}C_{x}\; p^{x}\; q^{n-x}$

$= {}^{3}C_{2} \left(\frac{1}{9}\right)^{2} \left(8/9\right)^{1}$

$= \frac{8}{243}$

Q, The probability of man hitting the target is $1/3$.

(i) If he fires five times, what is the probability of his hitting the target at least twice.

(ii) How many time must he fire so that the probability of his hitting the target atleast once is more than 90%

(i) $n = 5$, $p = 1/3$, $q = 2/3$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - \left[P(X=1) + P(X=0)\right]$$

In binomial distribution

$$\boxed{\begin{aligned} P(X=0) &= q^n \\ P(X=n) &= p^n \end{aligned}}$$ Always.

$$P(X \geq 2) = 1 - \left[\left(\tfrac{2}{3}\right)^5 + {}^5C_1 \left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right)^4\right]$$

$$= 1 - \left[\left(\tfrac{2}{3}\right)^4 \left[\tfrac{2}{3} + \tfrac{5}{3}\right]\right]$$

$$= \underline{\underline{\frac{131}{243}}}$$

ii) $P(X \geq 1) > 90\%$

$$1 - P(X=0) > 0.9$$

$$P(X=0) < 0.1$$

$$q^n < 0.1$$

$$\left(\frac{2}{3}\right)^n < 0.1$$

Taking log on both sides

Here asked for number of trials

$n = 5.$[illegible]

6 [illegible]

Q. If X and Y are the binomial r.v.

$X \sim B(2, p)$

$Y \sim B(4, p)$

If $P(X \geq 1) = 5/9$

Find $P(Y \geq 1) = ?$

$P(X \geq 1) = 5/9$

$1 - P(X=0) = 5/9$

$1 - (1-p)^n = 5/9$

$(1-p)^n = 4/9$

$q^n = 4/9$

Give $n = 2$

$q^2 = (1-p)^2 = 4/9$

$1-p = 2/3$

$p = 1/3$

For $Y \sim B(4, 1/3)$

$P(Y \geq 1) = 1 - P(Y=0)$

$= 1 - (q)^4$

$= 1 - (2/3)^4 = 1 - \frac{16}{81} = \frac{65}{81}$

Q, 2 dies are rolled 120 times. Find the average no: of times in which the number of the first die exceeds the no on the second die.

$n = 120$

$p = ?$

For finding p,

no: on first die exceeds the second die,

~~(2,1) (3,1) (~~

equal case → $6/36$, remaining $\frac{30}{36}$, Half will be first die > second die.

$\therefore \quad p = 15/36$.

$\therefore$ Average = Mean = $E[x] = np = 120 \times \frac{15}{36} = \underline{\underline{50}}$

Q, If x is a binomial r.v and $E(X) = 4$

$V(X) = 4/3$

Find (i) $P(x \leq 2)$

(ii) comment on β_1

i) $P(X \leq 2)$

$E(x) = 4$ $\qquad np q = \frac{4}{3}$

$np = 4$

$q = 1/3$ $\quad p = 2/3$ $\quad \frac{2n}{3} = 4$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \left(\frac{1}{3}\right)^6 + {}^6C_1 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right) + {}^6C_2 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2$$

$$= \frac{1}{3^6}\left[1 + 12 + 60\right] = \underline{73/729}$$

(ii) Given Data is -vely skewed since p value is more than $1/2$.

$P = 2/3 > 1/2$.

Q, If X is a binomial random variable, then find the value of

$$\sum_{x=0}^{n} \left(\frac{x}{n}\right) {}^nC_x \, p^x q^{n-x}$$

$$= \frac{1}{n} \sum_{x=0}^{n} x \, {}^nC_x \, p^x q^{n-x}$$

$$= \frac{1}{n} \boxed{\sum_{x=0}^{n} x \, P(x)}$$

$$= \frac{1}{n} \times \text{Mean}.$$

$$= \frac{1}{\cancel{n}} \times \cancel{n}P$$

$$= \underline{P}$$

POISSON DISTRIBUTION

Probability function is given by

$$\boxed{P(X;\lambda>0) = P(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!}, & \lambda>0,\ 0\le x<\infty \\ 0, & \text{otherwise.} \end{cases}}$$

→ It is used when observations are HIGH and success Probability is Low.

→ It is Used to Find RARE OCCURANCES.

→ poisson's equation is <u>time dependent</u> distribution.

→ ie, it is a Evolutionary Process.

Used to find Defect Probability.

Used to find Arrival Rate.

finition

If X is said to be poisson r.v defined in the interval $0<x<\infty$ with a parameter λ $(\lambda>0)$ and its probability mass function is

$$P(X;\lambda>0) = P(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!} & \lambda>0,\ 0\le x<\infty \\ 0 & \text{otherwice.} \end{cases}$$

Conditions

- → observations are infinitely large, $(n \to \infty)$
- → probability of success is very small $(p \to 0)$
- → $np = \lambda \Rightarrow p = \frac{\lambda}{n}$

Then $P(x : n, p) = \frac{e^{-np} (np)^x}{x!}$

It is approximation of binomial.

POISSON PROCESS

$$\boxed{P(x ; \lambda, t > 0) = \frac{e^{-\lambda t} (\lambda t)^x}{x!}}$$

PROPERTIES

1. $E(X) = \text{Mean} = \lambda$

$V(X) = \mu_2 = \lambda$

$\mu_3 = \lambda$

$\lambda > 0$

$\therefore \beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{1}{\lambda}$

ie, Poisson's distribution is always positively skewed.

It can never by symmetry

— positively skewed

2. MGF

$$M_x(t) = E\left[e^{tx}\right] = e^{\lambda(e^t - 1)}$$

3. Characteristic function.

$$\phi_x(t) = E\left[e^{itx}\right] = e^{\lambda(e^{it} - 1)}$$

te:

In ~~possi~~ poisson's ~~equ~~ distribution.

Mean = Variance = parameter λ.

It is always +vely skewed.

Sum of the independant poisson r.v's is also a oisson's r.v.

Difference b/w the independant poisson's r.v's is not a poisson random variable.

Q. A telephone switch board recieves 20 calls on an avg during an hour. Find the probability for a period of 5 min,

(i) No call is recieved.

(ii) Exactly 3 calls are recieved.

(iii) Atleast 2 calls are recieved.

reach
arrive
error
defect
} go for poisson's distribution.

Soln

avg = λ (for 60 min)

For 60 min $\rightarrow \lambda = 20$ calls

For 1 min $\rightarrow \frac{20}{60} = \frac{1}{3} = \lambda$

For 5 min = $\frac{1}{3} \times 5 = 1.65 = \lambda$

(i) $P(X=0) = \frac{e^{-1.65}(1.65)}{0!}$

$= e^{-1.65}$

(ii) $P(X=3) = \frac{e^{-1.65}(1.65)^3}{3!}$

(iii) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X=0) + P(X=1)] = 1 - \left[e^{-1.65} + \frac{1.65\, e^{-1.65}}{1!} \right]$

2, If X is a poisson r.v , then Find the Value of

$$\sum_{x=0}^{\infty} \left(\frac{x}{\lambda}\right) \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$\cancel{\frac{1}{\lambda} \times \lambda =}$$

$$\frac{1}{\lambda} \sum_{x=0}^{\infty} x \left(\frac{e^{-\lambda} \lambda^{x}}{x!}\right)$$

$$\frac{1}{\lambda} \times E(X)$$

$$= \frac{1}{\lambda} \times \lambda = \underline{\underline{1}}$$

NORMAL DISTRIBUTION (GAUSSIAN)

$$N(x;\mu,\sigma^2) = f(x) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} & \begin{array}{l} -\infty < x < \infty \\ -\infty < \mu < \infty \\ 0 < \sigma < \infty \end{array} \\ 0, \text{ otherwise.} \end{cases}$$

Definition : If X is said to be a normal r.v defined in the interval $-\infty < x < \infty$ with mean equal to μ and variance ~~is~~ equal to σ^2, Then the r.v is known as normal r.v. And its density function is

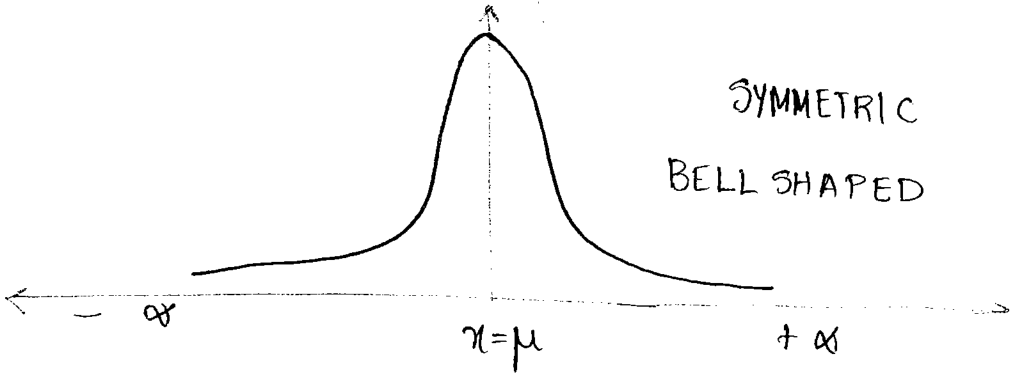
$$N(x;\mu,\sigma^2) = f(x) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} & \begin{array}{l} -\infty < x < \infty \\ -\infty < \mu < \infty \\ 0 < \sigma < \infty \end{array} \\ 0, \text{ otherwise.} \end{cases}$$

STANDARD NORMAL RANDOM VARIABLE

If X is a normal random variable with mean = 0 and variance = 1, then the random variable is known as standard normal random variable. It density function is

$$\boxed{N(0,1) = f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}}$$

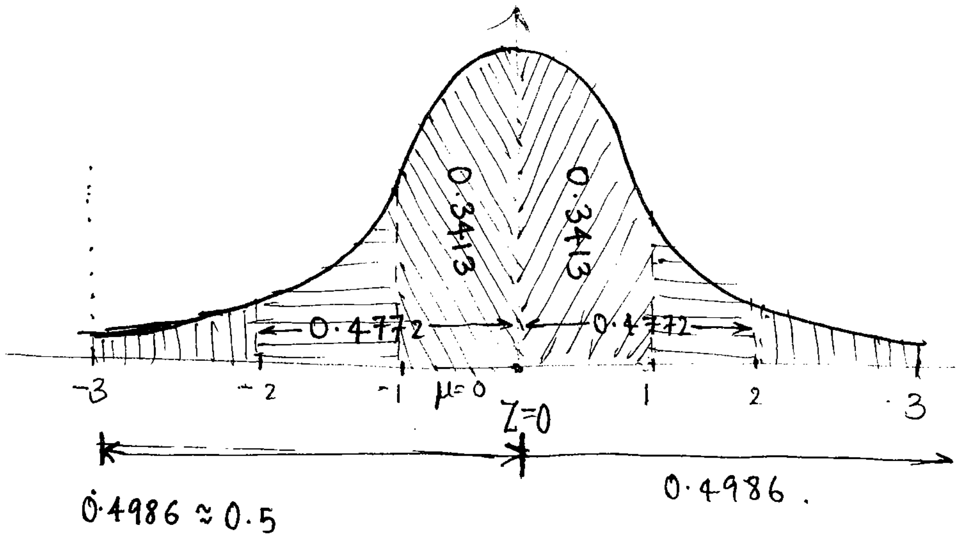
Mathematically a standard normal r.v is denoted by Z and is equal to $Z = \frac{X - E(X)}{\sqrt{V(X)}}$ & $\boxed{-3 \leq Z \leq +3}$



SYMMETRIC
BELL SHAPED

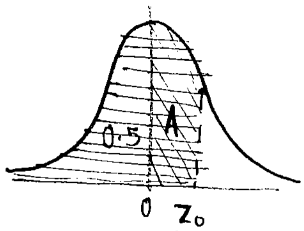
Normal distribution.

Standard Normal Distribution

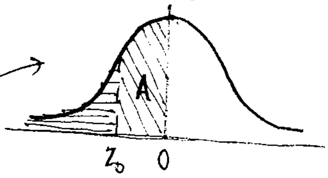


Areas under Normal curve

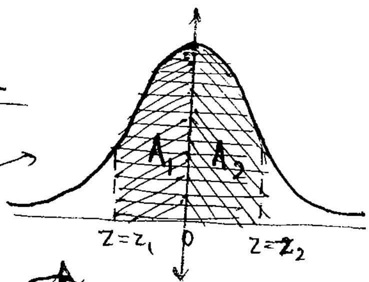
$P(Z \leq Z_0) = 0.5 + A \quad (Z_0 \text{ +ve}).$



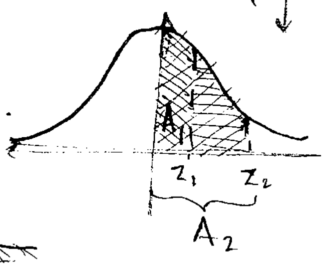
$P(Z \leq Z_0) = 0.5 - A \quad (Z_0 \text{ -ve}).$



$(Z_1 \leq Z \leq Z_2) = A_1 + A_2 \quad (Z_1 \text{ -ve and } Z_2 \text{ +ve})$



$P(Z_1 \leq Z \leq Z_2) = A_2 - A_2 \quad (Z_1 \text{ and } Z_2 \text{ +ve/-ve}).$



$P(Z \geq Z_0) = 0.5 + A \quad (Z_0 \text{ -ve})$

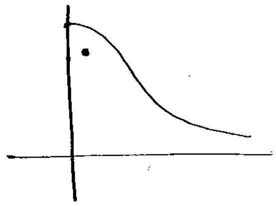
$\cancel{P(Z \geq Z_0)} = \cancel{0.5 - A} \quad (Z_0 \text{ +ve})$

Q, If X is normally distributed with mean = 20 and std deviation 3.33. Find the probability b/w $P(21.11 \leq x \leq 26.66)$. The area under the curve,

$Z_0 = 0$ to $Z = 0.33$ is 0.1293.

$$Z_1 = \frac{X_1 - \mu}{\sigma} \qquad Z_2 = \frac{X_2 - \mu}{\sigma}$$

$$Z_1 = \frac{1}{3} = 0.33 \qquad Z_2 = 2$$



$$P(21.11 \leq x \leq 26.66) = P(0.33 < Z < 2)$$

$$= 0.4772 - 0.1293$$

$$= \underline{\underline{0.3479}}$$

Q, If X is normally distributed with mean = 30 and std deviation is 5. Find $P(|x-30| > 5)$

$$P(|X-30| > 5) = 1 - P(|x-30| < 5)$$

$$\Rightarrow \cancel{(x-30)} \quad -5 < x-30 < 5$$

$$P(25 < x < 35)$$

$$Z_1 = \frac{25-30}{5} = \frac{-5}{5} = \underline{\underline{-1}}$$

$$Z_2 = \frac{35-30}{5} = \frac{5}{5} = \underline{\underline{1}}$$

$$P(-1 < Z < 1) = A_1 + A_2 = \underline{\underline{0.6826}}$$

2. A die is rolled 180 times. Using the normal distribution, find the probability that the face 4 will turn up atleast 35 times.

$$P(X \geq 35) = ?$$

We can use Binomial Distribution with $n = 180$, $p = 1/6$, $q = 5/6$.

$$E(x) = np = \frac{180}{6} = 30$$

$$V(x) = npq = \frac{180}{6} \times \frac{5}{6} = \overset{25}{\cancel{6 \times 5 = 30}}$$

$$Z = \frac{X - \mu}{\sigma} = \frac{35 - 30}{5}$$

$$= \underline{\underline{1}}$$

$$P(Z \geq 1) = 0.5 - 0.3413$$

$$= \underline{\underline{0.1587}}$$

Properties

$E(X) = \text{Mean} = \mu$

$V(X) = \text{Variance} = \mu_2 = \sigma^2$

$\mu_3 = 0 \Rightarrow \beta_1 = 0 \Rightarrow$ Symmetry.

MGF

$M_X(t) = e^{\left(t\mu + \frac{t^2\sigma^2}{2}\right)}$

(iv) characteristic

$$\phi_X(t) = e^{\left(it\mu - \frac{t^2\sigma^2}{2}\right)}$$

Properties of std Normal distribution.

→ $X \sim N(0,1)$

→ $E(X) = 0$

→ $V(X) = 1$

→ $M_X(t) = e^{t^2/2}$

→ $\phi_X(t) = e^{-t^2/2}$

→ Sum of the Independent normal r.variable is also a normal random variable.

→ The difference the independent normal r.v s is also a normal random Variable (Linear combination).

UNIFORM DISTRIBUTION [RECTANGULAR]

Definition: If X is a uniform r.v in the interval $a < x < b$ $(a<b)$ and it probabability density function is

$$\boxed{f(x) = \begin{cases} \dfrac{1}{b-a} & a<x<b \\ & a<b \\ 0 & \text{otherwise} \end{cases}}$$

MEAN $= \boxed{E(X) = \dfrac{a+b}{2}}$

VARIANCE $\boxed{V(X) = \dfrac{(b-a)^2}{12}}$

If X is uniform random variable in the interval $-a < x < a$ and its density function is

$$\boxed{f(x) = \frac{1}{2a}}$$

$\boxed{\text{Mean} = 0}$

$\boxed{\text{Variance} = \dfrac{a^2}{3}}$

The shape of uniform curve is rectangular.

A
$= 0.8461$
B
$= 1.1154$

CORRELATION / REGRESSION

CORRELATION

Karl pearson's Correlation.

The relation b/w the two dimensional r.v in bivariate data is known as regression (The degree of relation b/w the two variables is known as ~~regression~~ correlation.)

Types of correlation (i) Positive Correlation.

(ii) Negative Correlation.

Positive Correlation: If the changes in the both the variables are in the same direction (increasing or decreasing) Then those variables are known as positively correlated variables.

Negative Correlation: If the changes in the one variable is affecting the changes of other variable in reverse direction, then those variables are known as negatively correlated.

Karl pearson's Correlation eqn.

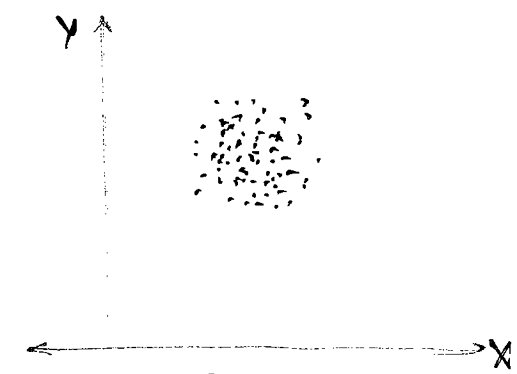
$$r(x,y) = \frac{Cov(x,y)}{\sqrt{x}\,\sqrt{y}}$$ where $Cov(x,y) = \frac{1}{n}\sum xy - \bar{x}\cdot\bar{y}$

$-1 \leq r \leq 1$

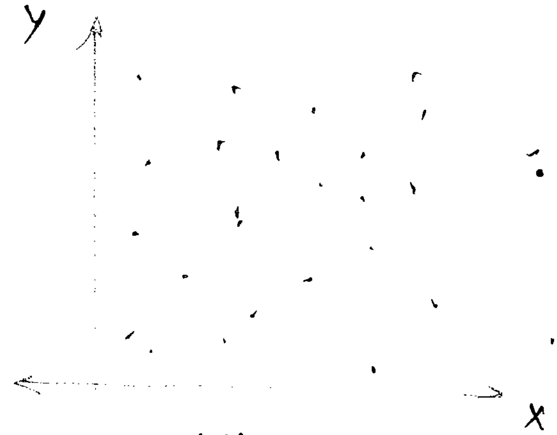
SCATTER DIAGRAM

It is a graphical representation of correlation. If the points are very close or thick on the XY plane then those points are correlated points.

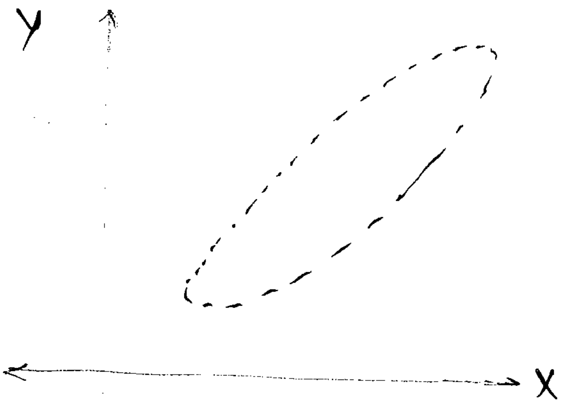
If the points are widely spreaded, then they are said to be uncorrelated.



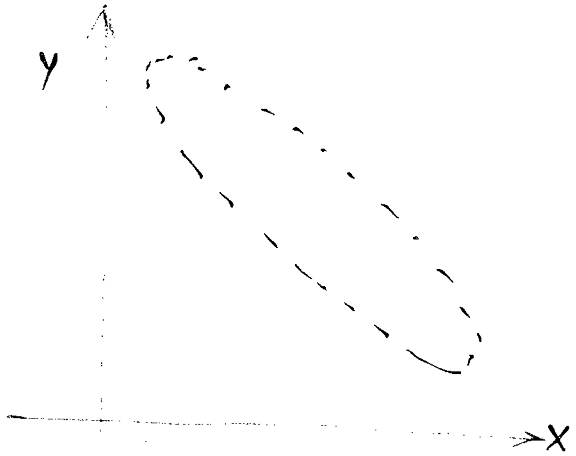
CORRELATED



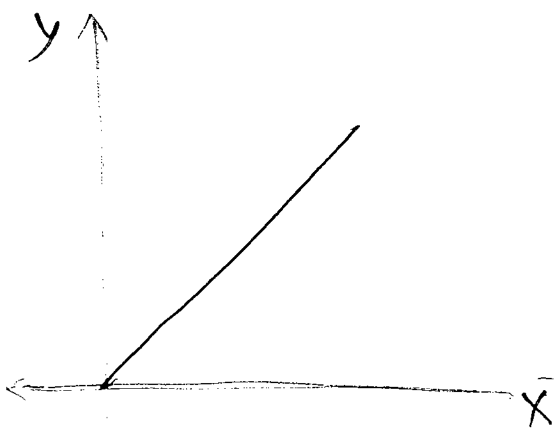
UNCORRELATED



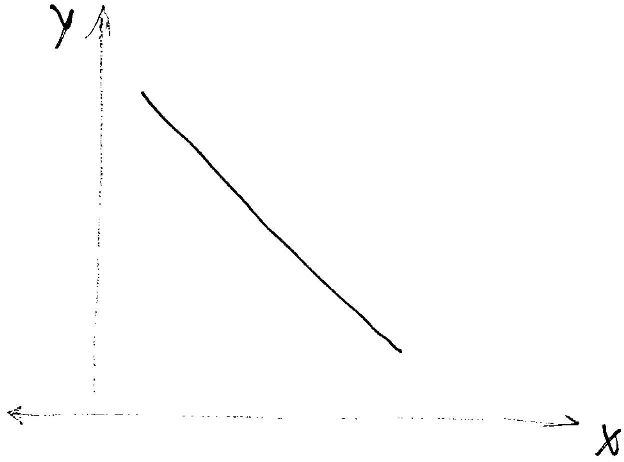
POSITIVELY CORRELATED



NEGATIVELY CORRELATED



PERFECT POSITIVELY



PERFECT NEGATIVELY

Note: → If X & Y are independent r.v's then covariance is zero.

i.e., $\boxed{Cov(X,Y) = 0 \implies r(X,Y) = 0}$

But converse of the statement is not true.

→ Correlation coefficient is independent of origin as well independent of change of scale.

REGRESSION → Y ON X, → X ON Y (Simple reggression)

Definition: The linear relationship b/w the 2-D random variable is known as regression.

LINES OF REGRESSION

$$Y - \bar{Y} = r \cdot \frac{\sigma_y}{\sigma_x}(X - \bar{X}) \qquad Y \text{ ON } X$$

$$X - \bar{X} = r \cdot \frac{\sigma_x}{\sigma_y}(Y - \bar{Y}) \qquad X \text{ ON } Y$$

$$\left.\begin{aligned} \text{where } r \cdot \frac{\sigma_y}{\sigma_x} &= b_{YX} \\ \& \ r \cdot \frac{\sigma_x}{\sigma_y} &= b_{XY} \end{aligned}\right\} \text{Regression Coefficient.}$$

→ Correlation coefficient is the Geometric mean b/w regression coefficients.

$$\boxed{r = \pm\sqrt{b_{yx}\, b_{xy}}}$$

Note: Both the regression coefficients must have a same sign.

ie, If both +ve $\Longrightarrow$ r is +ve.
If both −ve $\Longrightarrow$ r is −ve

If $b_{yx} > 1 \Rightarrow b_{xy} < 1$ (Vice versa).

If the regression coefficients are equal, their variances also equal.

$\boxed{b_{xy} = b_{yx}} \Rightarrow r \cdot \frac{\sigma_x}{\sigma_y} = r \cdot \frac{\sigma_y}{\sigma_x}$

$$\Rightarrow \boxed{\sigma_y^2 = \sigma_x^2}$$

Regression equations are passes through the point $\bar{X}, \bar{Y}$

Regression Coefficient is independent of change of origin as well as dependent of change of scale.

Angle b/w Regression lines

$$\boxed{\theta = \tan^{-1}\left(\frac{1-r^2}{|r|} \cdot \frac{\sigma_x \sigma_y}{(\sigma_x)^2 + (\sigma_y)^2}\right)}$$

$r = 0 \Rightarrow \theta = \pi/2$

$r = 1 \Rightarrow \theta = 0$ or π

Q, The regression equation are $3x + 2y = 1$.
$2x + 4y = 0$.

(i) Find r?

(ii) $\bar{X}, \bar{Y}$

whichever the coefficient in the expression is higher then it is the DEPENDENT VARIABLE.

Consider $3X + 2Y = 1$

Dependent variable

$\therefore$ X on Y

X on Y	Y on X
$3X + 2Y = 1$	$2x + 4y = 0$
$3X = 1 - 2Y$	$4Y = -2X$
$X = 1/3 - 2/3\,Y$	$Y = -2/4\,X$
$b_{xy} = -2/3$	$b_{yx} = -1/2$

— Both −ve Hence r also — −ve.

$$r = \sqrt{\cdot 2/3 \times 1/2}$$

$$r = -\sqrt{1/3}$$

$$r = -1/\sqrt{3}$$

(ii) Means $\bar{x}$ & $\bar{y}$ satisfies eqn.

$$6\bar{x} + 4\bar{y} = 2$$

$$2\bar{x} + 4\bar{y} = 0$$

$$4\bar{x} = 2$$

$$\bar{x} = 1/2$$

$$\bar{y} = -1/4$$

$\therefore$ Means $(\bar{x}, \bar{y}) = (1/2, -1/4)$

; $x - 2y = 2$

$3x - y = 1$

(i) r (ii) $\bar{x}, \bar{y}$

Y on X

$x - 2y = 2$

$x - 2 = 2y$

X on Y

$3x = 1 + y$

$x = 1/3 + y/3$

$$r = \sqrt{\tfrac{1}{2} \times \tfrac{1}{3}}$$

$$r = \frac{1}{\sqrt{6}}$$

(ii) $$\bar{x} - 2\bar{y} = 2$$

$$3\bar{x} - \bar{y} = 1$$

$$3\bar{x} - 6\bar{y} = 6$$

$$3\bar{x} - \bar{y} = 1$$

$$-5\bar{y} = 5$$

$$\bar{y} = -1$$

$$3\bar{x} + \cancel{1} = \cancel{1}$$

$$\bar{x} = 0$$

$$(\bar{x}, \bar{y}) = (0, -1)$$